The use of machine learning and grey-box models to solve complex time-consuming RBDO problems

Application to mass production mechanical systems

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Objectives

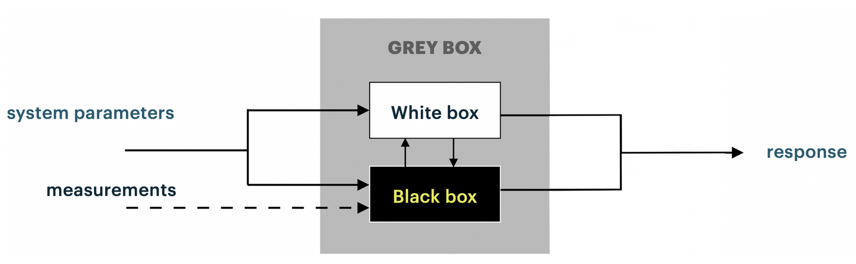
1. Develop an efficient approach for reliability estimation
2. Apply this method in a multi-fidelity modelling context
3. Apply zero-order optimization problem based on machine learning separators
4. Apply developments to production processes in collaboration with Radiall

Introduction

Context

- Structural design goal: to be optimal, reliable regarding uncertainties
- Applying grey-box approaches for reliability analysis, optimizing and controlling of production process and systems

Grey-box modelling



- **white box**: physics-based computational models
- **black box**: mathematical models based on ML approaches built from observational data
- **grey box**: fusing information to relax the need to exactly model the underlying physics, while requiring considerably less data

Methodology

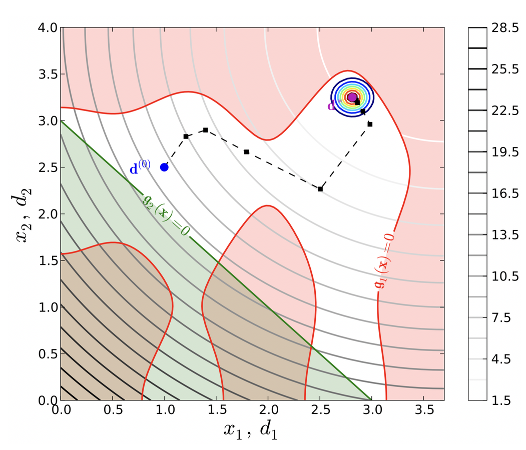
RBDO formulation

- Optimization under reliability constraints: aim to identify admissible design with optimal performance
- Minimizing a cost function f while satisfying the performance function g
- Optimal solutions lie on the boundaries of the admissible space

$$\text{Find:} \quad \bar{X}_{OptRel} = \arg\min_{\bar{X}} f\left(\bar{X}, P^{(k)}\right)$$

$$\text{Subject to:} \quad Prob\Big(g\big(X(\bar{X}, \omega), P(\omega)\big) \leqslant 0\Big) \leqslant P_{target}$$

- **Main problem**: computational time consuming
  - ↳ Metamodel-based strategy → Adaptive Kriging
  - ↳ Classify a MC sample using ML separators defined in an augmented-space and 0-order algorithms (e.g. Genetic Algorithm)



First investigation: review on Python toolboxes for Kriging

1. Focus on:
   - ↳ comparing the various settings available for each library
   - ↳ to ascertain how they perform and differ under similar assumptions
2. Comparison on:
   - ↳ computational time-cost for different size of ED
   - ↳ prediction accuracy by means of Mean of the Squared Errors (MSE)

$$MSE = \frac{1}{n}\sum_{i=1}^{n}\left(Y(x_i^*) - \tilde{Y}(x_i^*)\right)^2$$

Benchmark case-study

- FE model: rectangular shell plate (1.5m×1m) clamped at the four edges
- Load: pressure of 100 Pa uniformly applied at the surface
- Fiber orientation: $x = \{x_1, \ldots, x_m\}$ where $m \in [2, 4, 8, 16, 32]$
- MC sampling: input space with a uniform distribution between 0° and 180°
- QoI: displacement Y at the center of the plate
- Kriging surrogate: $\tilde{Y}(x) = \sum_{j=1}^{p} \beta_j f_j(x) + Z(x)$
- Matérn 3/2 kernel: $R(x, x'; \theta) = \left(1 + \sqrt{3}\frac{|x-x'|}{\theta}\right) exp\left[-\sqrt{3}\frac{|x-x'|}{\theta}\right]$

Benchmark case-study results

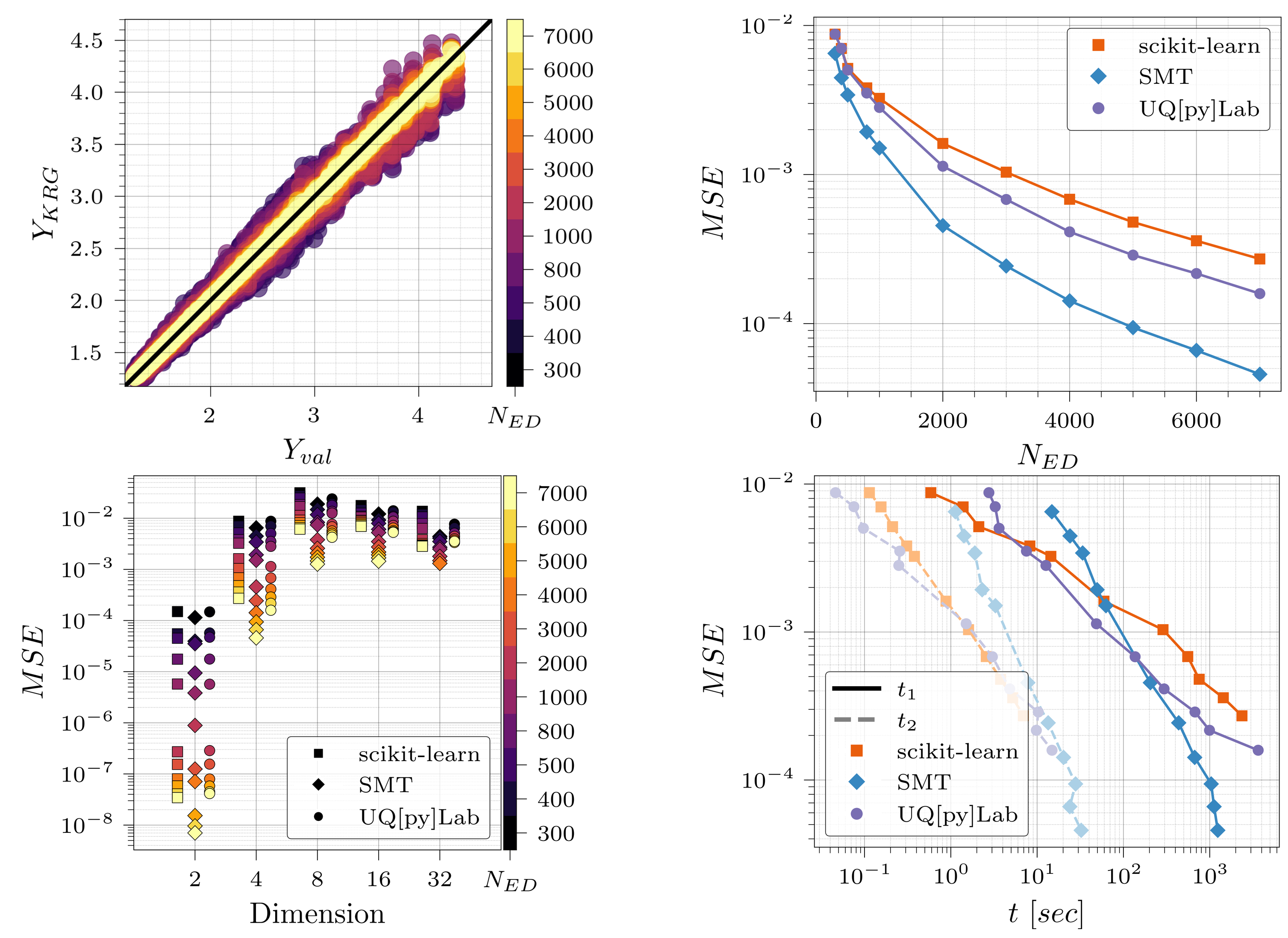


Figure 1: North-West: Cross-validation plot for SMT, dimension 4; North-East: MSE for increasing value of N_{ED}; South-West: MSE at different dimensionality; South-East: MSE vs time.

Industrial applications

Applications to product cost reduction in mass production

Discussion and future work

- Discrepancies have been observed among various Python packages
- The review will be extended to more packages and scenarios (e.g. different kernel types and other optimization algorithms)
- Zero-order algorithm will be investigated to deal with RBDO problems
- Multifidelity computer codes with different confidence levels will be investigated and applied to decrease the global optimization time

References

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Acknowledgments

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Skłodowska-Curie grant agreement No. 955393.

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